LIMITS AND CONTINUITY

Problem 1 (jS, Exercises 14.2(10,13,16,19,22))**.** *Find the limit, if it exists, or show that the limit does not exists.*

(1) $\lim_{(x,y)\to(0,0)} \frac{5y^4 \cos^2 x}{\sqrt{x^4+y^4}}$.

(2) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$.

(3) $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4}$.

(4) $\lim_{(x,y,z)\to(\pi,0,1/3)} e^{y^2} \tan(xz)$.

(5) $\lim_{(x,y,z)\to(0,0,0)} \frac{x^2 y^2 z^2}{x^2+y^2+z^2}$.

Problem 2 (jS, Exercises 14.2(39,40,41))**.** *Use polar coordinate to find the following limits.*

(1) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$.

(2) $\lim_{(x,y)\to(0,0)} (x^2 + y^2) \ln(x^2 + y^2)$.

(3) $\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$.

Problem 3 (jS, Exercises 14.2(30,35,36))**.** *Determine the set of points at which the following functions are continuous.*

(1) $f(x,y) = \cos(\sqrt{1+x-y})$.

(2) $f(x,y,z) = \arcsin(x^2 + y^2 + z^2)$.

(3) $f(x,y,z) = \sqrt{y-x^2} \ln z$.

Problem 4 (jS, Exercise 14.2.45)**.** *Show that the function $f(x) = |x|$ is continuous in $\mathbb{R}^n$. [Hint: Consider $|x-a|^2 = (x-a) \cdot (x-a)$.]*

REFERENCES

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.